CORE-COLLAPSE TIMES OF TWO-COMPONENT STAR CLUSTERS

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Abstract

We examine the corecollapse times of isolated, two-mass-component star clusters using Fokker-Planck models. With initial condition of Plummer models, we find that the corecollapse times of clusters with $M_1/M_2 \gg 1$ are well correlated with $(N_1/N_2)^{1/2}(m_1/m_2)^2 t_{rh}$, where M_1/M_2 and m_1/m_2 are the light to heavy component total and individual mass ratios, respectively, N_1/N_2 is the number ratio, and t_{rh} is the initial half-mass relaxation time scale. We also find two-component cluster parameters that best match multi-component (thus more realistic) clusters with power-law mass functions.

Subject headings: celestial mechanics, stellar dynamics — globular clusters : general

1. INTRODUCTION

The course of dynamical evolution of pre- and post-collapse globular clusters is determined by many factors such as initial mass function, nature and efficiency of energy generation mechanisms, tidal cut-off, anisotropy of velocity distribution, initial population of binaries, and stellar evolution. There have been many efforts in developing more and more complex cluster models including such factors, making analysis and interpretation rather difficult. To study the dynamical evolution of globular clusters more realistically, among others, Chernoff & Weinberg (1990) included the effects of stellar evolution, Lee, Falhman, & Richer (1991) used multi-component models, Takahashi (1995) includeded an anisotropic velocity distribution.

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However, studying simpler models could be more instructive in identifying important physical processes governing the evolution. Kim, Lee, & Goodman (1997; hereafter KLG) studied on the postcollapse evolution of cluster variables and the gravitational oscillation using two-component Fokker-Planck models. In this paper, as a supplementary study to KLG, we present a fitting formula for the corecollapse times of two-component models and compare the results of two- and multi-component models. As in KLG, here both tidal-capture binary heating and tree-body binary heating are included, and clusters are assumed to be isotropic and isolated (no tidal cutoff). For the methods that we are using here and the benefits of studying simpler models (two-component models), readers are referred to KLG and references therein.

Corecollapse times of two-component clusters were presented by Inagaki & Wiyanto (1984), Inagaki (1985), and Lee (1995) among others. These papers calculated corecollapse times as a function of M_2/M_1, the ratio of total masses of heavy component to light component, and found that the ratio of corecollapse time to initial half-mass relaxation time, t_{cc}/t_{rh}, has a minimum value at $M_2/M_1 \sim 0.1$. However, the paramter M_2/M_1 may be divided into m_2/m_1, the individual mass ratio, and N_2/N_1, the number ratio. In the present paper, we calculate the corecollapse times of two-component models as a fuction of more complete two-copmonent cluster parameters, and find a fitting formula between them. However, the clusters studied here are restricted to those with $M_1 \gg M_2$ as in KLG.

On the other hand, it would be helpful in interpreting the results of two-component models if the similarities and discrepancies between the results of two- and multi-component models are well known. In the present paper, we also compare two-component models to 11-component models, and thus provide a way to extrapolate to more realistic cluster the results of two-component clusters such as those in KLG.

2. CORE-COLLAPSE TIMES

To calculate the corecollapse times of two-component star clusters, we have performed total 11 runs of direct numerical integration of the orbit-averaged Fokker-Planck equation with a local approximation. The code used here is descended from Cohn (1980). Parameters of our two-component runs are shown in Table 1. This set of parameters has been chosen such

that it provides all possible combinations of parameters M, N, m_2/m_1, and N_1/N_2, where M is the cluster mass and N is the total number. Note that in all our runs, the total mass of heavy component, M_2, is negligible compared to the total mass of light component, M_1, and thus $m_1 \approx M/N_1$. The initial density and velocity profiles are given by Plummer models with $v_{c1}/v_{c2} = 1$ and $\rho_{c1}/\rho_{c2} = M_1/M_2$, where v_c^2 is the three-dimensional core velocity dispersion, and ρ_c is the core density.

Corecollapse times of our runs are shown in Table 1 in units of 10^{10} yr and t_{rh}. We empirically found that t_{cc} can be fitted by the following formula:

$$t_{cc} \approx 4.2 \times 10^9 \mathrm{yr} \left(\frac{N_1}{N_2}\right)^{1/2} \left(\frac{m_1}{m_2}\right)^2 N_5 M_5^{-1/2} \left(\frac{r_h}{5\,\mathrm{pc}}\right)^{3/2}, \qquad (1)$$

where $N_5 \equiv N/10^5$, $M_5 \equiv M/10^5\, M_\odot$, and r_h is the initial half-mass radius. Each t_{cc} value is plotted over the righthand side of the above equation in Figure 1, which shows a good X-Y correlation. Equation (1) is to be compared with the standard half-mass relaxation time scale,

$$t_{rh} \equiv \frac{v_m^2}{\langle v_\parallel^2 \rangle_{v=v_m}} = \frac{M^{1/2} r_h^{3/2}}{6.7 G^{1/2} m \ln 0.4N}, \qquad (2)$$

where v_m is the root-mean-square three-dimensional velocity of the whole cluster and $\langle v_\parallel^2 \rangle_{v=v_m}$ is the average change of v_m^2 in parallel component to initial v_m per unit time.

Isolated single-mass clusters with initial condition of Plummer models collapse at 15.4 t_{rh} (Cohn 1980), where t_{rh} is the half-mass relaxation time scale and does not vary much until the corecollapse takes place. However, the ratios of the time required for corecollapse t_{cc} to t_{rh} and core relaxation time scale t_{rc} strongly depend on the density and velocity profiles. Quinlan (1996) found that for single-mass clusters, t_{cc} varies much less when expressed in units of t_{rc} divided by a dimensionless measure of the temperature gradient in the core. Although in single-mass clusters the velocity profile (as well as other physical parameters) evolves by the two-body relaxation, two- or multi-component clusters have another driving force: the equipartition.

Both mass segregation and equipartition are envolved in determination of the time to corecollapse,

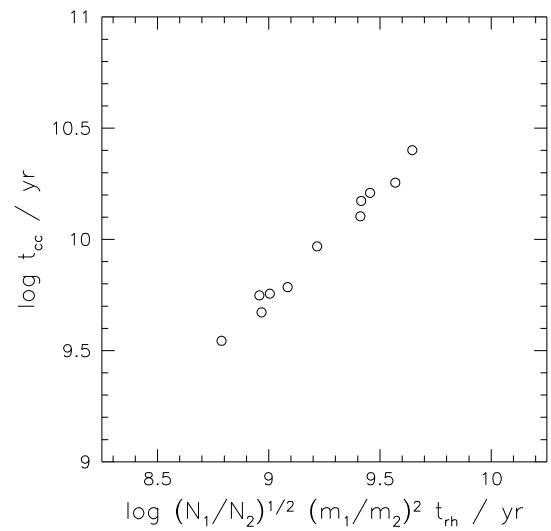

Fig. 1.— Corecollapse times of runs in Group A. $N_5 \equiv N/10^5$ and $M_5 \equiv M/10^5\, M_\odot$.

and this complexity makes the theoretical interpretation of the above correlation between t_{cc} and cluster parameters quite difficult. Here we suggest the following analysis as one way to explain this correlation.

The actual duration of corecollapse is very small compared to the time to corecollapse from the begining of cluster's evolution. Instead, clusters spend most of their precollapse phases under mass segregation process and approach to the onset of homologous phase of corecollapse. If a considerable degree of equipartition is accomplished in the precollapse phase as in all of our two-component models, the time to the onset of corecollapse will be determined by how fast the light component gains the energy from the heavy component via equipartition. Thus one may define the precollapse time scale of two-component clusters t_{r2} as following:

$$t_{r2} \equiv \frac{v_{m1}^2}{\langle v_{\parallel 1}^2 \rangle_2}, \qquad (3)$$

where $\langle v_{\parallel 1}^2 \rangle_2$ is the velocity dispersion change of the light component via interactions with heavy component. Using the standard expression for the average velocity dispersion change per unit time, one has

$$\langle v_{\parallel 1}^2 \rangle_2 \propto \frac{G^2 m_2 \rho_{m2}}{v_{m2}}, \qquad (4)$$

Table 1
Parameters and Core-Collapse Times of Two-Component Models

Run	$\frac{m_2}{m_1}$	$\frac{N_1}{N_2}$	M $(M_\odot)$	N	m_2 $(M_\odot)$	t_{cc} $(10^{10}\mathrm{yr})$	t_{cc}/t_{rh}
baab	2	100	10^5	141457	1.4	1.27	12.42
caab	3	100	10^5	210125	1.4	0.93	6.34
faab	4	100	10^5	277473	1.4	0.61	3.23
cdab	3	30	10^5	201299	1.4	0.56	3.97
cbab	3	300	10^5	212871	1.4	1.62	10.92
caab1	3	100	10^5	70042	3×1.4	0.35	6.47
caab2	3	100	10^5	630374	$\frac{1}{3}\times1.4$	2.52	6.28
baab3	2	100	10^5	212185	$\frac{2}{3}\times1.4$	1.80	12.17
faab3	4	100	10^5	208104	$\frac{4}{3}\times1.4$	0.47	3.23
caeb	3	100	3×10^4	63037	1.4	0.57	6.34
cabb	3	100	3×10^5	630374	1.4	1.49	6.43

Note.—The initial half-mass radii r_h of these runs are all 5 pc.

where the heavy component mean density ρ_{m2} is proportional to M_2/r_h^3 and the Coulomb logarithm has been omitted. It is also assumed that $v_{m1} \sim v_{m2}$. Spitzer (1969, 1987) showed that for a two-component cluster of polytropic index n between 3 and 5 with $M_1 \gg M_2$ and a Maxwellian velocity distribution in a parabolic potential well, the minimum degree of the global equipartition is a function of cluster's parameters such that

$$\left.\frac{m_2 v_{m2}^2}{m_1 v_{m1}^2}\right|_{\min} \propto \left(\frac{N_2}{N_1}\right)^{2/3}\left(\frac{m_2}{m_1}\right)^{5/3}. \qquad (5)$$

With the minimum value of equation (5) and assumptions that $M_1 \gg M_2$ and $v_{m1}^2 \sim GM/r_h$, equation (3) now becomes

$$\begin{aligned} t_{r2} &\propto \left(\frac{N_1}{N_2}\right)^{2/3}\left(\frac{m_1}{m_2}\right)^{5/3} N M^{-1/2} r_h^{3/2} \\ &\propto \left(\frac{N_1}{N_2}\right)^{2/3}\left(\frac{m_1}{m_2}\right)^{5/3} t_{rh}. \end{aligned} \qquad (6)$$

The above is in the same form as equation (1) with only small discrepancies in the exponents. Although equation (5) has been used for derivation of the above equation, we find that the degrees of equipartition in the precollapse phases of our two-component runs do not directly correlate with the minimum values of equation (5). In fact, exact equipartition is usually not accomplished even when the value of equation (5) is less than unity, because as mass segregation of heavy component progresses, interactions between heavy and light components occur less. Thus equation (5) should be regarded as a degree of tendancy to equipartition and it is this tendancy that t_{r2} requires in its definition.

While Quinlan (1996) introduced a temperature gradient in the core in a derivative form to explain a huge variation in t_{cc} for clusters with different initial profiles, here we introduced both density and velocity gradients of heavy component naturally into the time scale by considering global equipartition.

3. COMPARISON WITH MULTI-COMPONENT CLUSTERS

Clusters have continuous mass functions. However, mass functions are usually realized with discrete mass components in numerical calculations. Scientists found that 10 to 20 components are enough

to represent continuous mass functions for Fokker-Planck models, and such numerical representation for a given mass function is quite straightforward for these multi-component clusters: there is only a question of choice of each component's mass bin and a representative value. However, when the number of components is reduced to 2 for the sake of analytical simplification, such choice is not so simple because dynamically important mass and corresponding number of stars may be different from simple mean mass and total number of a certain mass range. Therefore two-component cluster parameters (such as m_2/m_1, N_1/N_2, and N) that well represent a continuous mass function should be numerically found through comparisons of the evolution of two- and multi-component clusters.

In this section we will compare our multi-component models with the two-component models in KLG varying M and the mass function of the multi-component models. Cluster parameters of our multi-component models are given in Table 2. The initial density and velocity profiles are given by Plummer models. The initial half-mass radii of all multi- and two-component models are 5 pc. The number of component is 11 and we adopt a power-law mass function:

$$N(m)dm \propto m^{-(x+1)}dm, \tag{7}$$

where x is the mass spectral index and the Salpeter mass function has $x = 1.35$. For a bin i with boundaries m_{ia} and m_{ib}, the total mass in the bin is obtained by

$$M_i = \int_{m_{ia}}^{m_{ib}} mN(m)dm. \tag{8}$$

Then the number of stars in the bin is $N_i = M_i/m_i$, where m_i is the representative mass of each bin. The main-sequence star mass range was selected to be $0.08\,M_\odot - 0.8\,M_\odot$. Following Sigurdsson & Phinney (1995), the stars of initial mass m_i between $0.8\,M_\odot$ and $4.7\,M_\odot$ were assumed to have evolved to white dwarfs of mass $0.58 + 0.22 \times (m_{\rm MS} - 1.0)\,M_\odot$, where $m_{\rm MS}$ is the main-sequence mass, while stars of $m_{\rm MS}$ between $4.7\,M_\odot$ and $8.0\,M_\odot$ were assumed to disrupt completely. The stars heavier than $8.0\,M_\odot$ but lighter than $15.0\,M_\odot$ were assumed to become neutron stars of mass $1.4\,M_\odot$. Neutron stars are born with a kick velocity due to an asymmetric explosion, and they are ejected from the cluster if the kick velocity is greater than the escape velocity of the cluster. However, we assumed that all neutron stars remain in the cluster, because the retention rate of neutron stars are not well known and the precise realization of real clusters is not our goal in this study. The mass range, representative mass, and number of stars of each component is shown in Table 3. Our multi-component models include both three-body binary heating and tidal-capture binary heating, but we find that the postcollapse phases of all our runs are driven by three-body binary heating.

We find a two-component model which best describes a given multi-component model by comparing the values of cluster variables ρ_c, v_c, r_h at $t = 10^{11}$ yr, and t_{cc}. An epoch of 10^{11} yr has been selected as in KLG because by that time, our runs have reached self-similar expansion phase. With two-component models, KLG found the following numerical values:

$$\rho_c \simeq 4.5 \times 10^5 \ M_\odot/\mathrm{pc}^3 \left(\frac{m_2}{m_1}\right)^{-10/3} N_5^{10/3} t_{11}^{-2.0}; \tag{9a}$$

$$v_c \simeq 3.8\,\mathrm{km/s} \left(\frac{m_2}{m_1}\right)^{-1/2} N_5^{1/3} M_5^{1/3} t_{11}^{-0.32}; \tag{9b}$$

$$r_c \simeq 0.042\,\mathrm{pc} \left(\frac{m_2}{m_1}\right)^{7/6} N_5^{-4/3} M_5^{1/3} t_{11}^{0.65}; \tag{9c}$$

$$r_h \simeq 35\,\mathrm{pc}\ N_5^{-2/3} M_5^{1/3} t_{11}^{0.65}, \tag{9d}$$

where $N_5 \equiv N/10^5$, $M_5 \equiv M/10^5\,M_\odot$, and $t_{11} \equiv t/10^{11}\,\mathrm{yr}$. On the other hand, the numerical values from our multi-component models are given in Table 2. There are four two-component parameters to be determined for a given multi-component model, m_2/m_1, N_1/N_2, N, and M. However, since cluster variables at a certain epoch in the postcollapse phase are independent of N_1/N_2 as in equation (9), N_1/N_2 has to be determined from the corecollapse time, equation (1). Then the rest three parameters, m_2/m_1, N, and M, may be determined from equation (9). This method will be called Method A, and parameters obtained in this way are given in Table 4.

With Method A, our multi-component model B2 is best described by a two-component model with $m_2/m_1 = 2.3$, $N_1/N_2 = 29$, $N = 1.4 \times 10^5$, and $M = 0.97 \times 10^5\,M_\odot$. Note that with these parameters, $m_2 \simeq 1.6\,M_\odot$, which is little higher than the mass of the heaviest component of our multi-component models, $1.4\,M_\odot$. Neutron stars play an important role in dynamical evolution of globular clusters: a considerable fraction of dynamical binary formation (as apposed to primordial binaries) envolves neutron stars. For this reason, in two-component clusters, the heavy component is often targeted for neutron stars and the light component is for main-sequence stars. Thus in finding the best matching two-component parameters, it could be more meaningful if m_2 is set to

Table 2
Parameters and Results of Multi-Component Models

Run	x	M ($M_\odot$)	N	t_{cc} (10^{10}yr)	Values at $t = 10^{11}$ yr: ρ_c ($M_\odot$ pc^{-3})	v_c (km s^{-1})	r_h (pc)
A2	1.00	10^5	402857	0.528	7.63×10^4	2.79	32.8
B2	1.35	10^5	509201	0.584	8.13×10^4	2.75	27.7
C2	1.50	10^5	552232	0.621	8.19×10^4	2.73	26.2
B1	1.35	3×10^4	152760	0.356	1.71×10^3	1.30	38.5
B3	1.35	3×10^5	1527603	0.931	2.58×10^6	5.44	20.7

Note.—The initial half-mass radii r_h of these runs are all 5 pc.

Table 3
Mass Spectra of Multi-Component Models

Bin	m_i ($M_\odot$)	Mass Range ($M_\odot$)	$x = 1.00$: N_i	$x = 1.00$: $m_i \cdot N_i$	$x = 1.35$: N_i	$x = 1.35$: $m_i \cdot N_i$	$x = 1.50$: N_i	$x = 1.50$: $m_i \cdot N_i$
1	0.1	0.08 – 0.15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.2	0.15 – 0.25	0.40631	0.81262	0.33263	0.66526	0.30517	0.61034
3	0.3	0.25 – 0.35	0.17842	0.53526	0.12584	0.37752	0.10826	0.32478
4	0.4	0.35 – 0.45	0.09995	0.39980	0.06359	0.25436	0.05223	0.20892
5	0.5	0.45 – 0.55	0.06385	0.31925	0.03753	0.18765	0.02985	0.14925
6	0.6	0.55 – 0.65	0.11712	0.70272	0.05766	0.34596	0.04264	0.25584
7	0.7	0.65 – 0.80	0.09010	0.63070	0.04104	0.28728	0.02945	0.20615
8	0.9	0.80 – 1.0	0.02428	0.21852	0.00822	0.07398	0.00516	0.04644
9	1.1	1.0 – 1.2	0.01286	0.14146	0.00389	0.04279	0.00232	0.02552
10	1.3	1.2 – 1.4	0.00774	0.10062	0.00215	0.02795	0.00124	0.01612
11	1.4	1.4 – 1.4	0.00913	0.12782	0.00182	0.02548	0.00091	0.01274

Note.—N_i and $m_i \cdot N_i$ are normalized with bin 1 values.

Table 4
Best Matching Two-Component Cluster Parameters

Run	Method A $\frac{m_2}{m_1}$	$\frac{N_1}{N_2}$	N_5	M_5	Method B $\frac{m_2}{m_1}$	$\frac{N_1}{N_2}$	N_5	M_5	N_2
A2	1.78	13.1	1.05	0.90	1.70	10.9	1.00	0.82	8400
B2	2.33	28.5	1.40	0.97	2.17	21.3	1.30	0.84	5800
C2	2.56	39.4	1.54	0.99	2.36	28.4	1.41	0.84	4800
B1	2.74	53.9	0.51	0.35	2.37	30.0	0.44	0.26	1400
B3	2.05	18.1	3.47	2.49	2.00	16.3	3.38	2.36	20000

Note.—$N_5 \equiv N/10^5$ and $M_5 \equiv M/10^5\,M_\odot$. N_2 has been approximately calculated by $N/(N_1/N_2+1)$ and has only two significant digits.

$1.4\,M_\odot$. With a restriction of $m_2 = 1.4\,M_\odot$, now the number of variables in equation (9) required for determination of two-component cluster parameters is reduced to two. Since ρ_c and r_h are two cluster variables that represent the status of the core and envelope, respectively, we use these variables along with $m_2 = 1.4\,M_\odot$ and equation (1) for our second method (Method B) to find the best matching two-component model (see Table 4).

With Method B, model B2 is now best described by a two-component model with $m_2/m_1 = 2.2$, $N_1/N_2 = 21$, $N = 1.3 \times 10^5$, and $M = 0.84 \times 10^5\,M_\odot$. Note that with these parameters, $N_2 = 4890$ and this value is about the same with the number of stars in the heaviest four bins (bins only for degenerate stars) of model B2 ($\sum_{i=8,11} N_i = 5176$). This may imply that the epoch of corecollapse is mainly determined by the number of stars above the turnoff mass. This fact also holds for other runs with different M and x.

For clusters with $N \propto M$ (as for our multi-component clusters B1, B2, and B3), equation (9) may be written as $\rho_c \propto M^{10/3}$, $v_c \propto M^{2/3}$, and $r_h \propto M^{-1/3}$. From runs B1, B2, and B3 in Table 2, ρ_c, v_c, and r_h are found to be proportional to $M^{3.18}$, $M^{0.622}$, and $M^{-0.269}$, respectively. The absolute values of these exponents are little smaller than equation (9). However, since the discrepancies are not so significant, we conclude that the evolution aspects of the postcollapse multi-component clusters are still well predictable from the numerical and analytical results of two-component clusters. On the other hand, cluster variable values at $t = 10^{11}$ yr show relatively small x dependence.

The results from Method B in Table 4 indicate that multi-component clusters may be described by two-component clusters with masses 15 to 20 % less and with m_1 near the turnoff mass. Of course, this m_1 is dependent on x such that clusters with steeper mass function are matched by two-component clusters with smaller m_1. However, interestingly, lighter multi-component clusters *with the same* x also require smaller m_1. This comes from the fact that $N \propto M$ holds for runs B1, B2, and B3, while not for their matching two-component clusters by Method B ($N \propto M^{0.92}$). For Method A, the best matching two-component clusters of runs B1, B2, and B3 show $N \propto M^{0.98}$, which results in nearly the same m_1. Thus we conclude that the above difference in m_1 values by Method B for clusters with the same x stems from the restriction, $m_2 = 1.4\,M_\odot$.

The evolution of the two-component model with the above parameters is plotted in Figure 2 as well as that of model B2. Cluster variables ρ_c, v_c and r_h of two runs well coincide. Only the corecollapse times show a small discrepancy. This is partly because of the dispersion of t_{cc} from the fitting formula, equation (1), and partly because of the small N_1/N_2 value: equation (1) is to be used for clusters with $M_1 \gg M_2$.

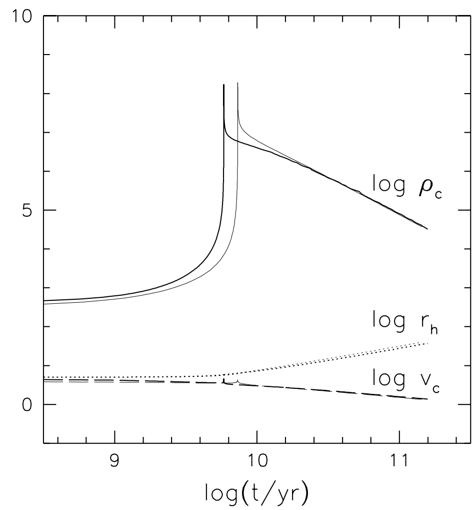

Fig. 2.— Comparison of the evolution of multi-component model (run B2; thick lines) and best-matching two-component model by Method B ($m_2/m_1 = 2.2$, $N_1/N_2 = 21$, $N = 1.3 \times 10^5$, and $M = 0.84 \times 10^5\,M_\odot$; thin lines). The units of ρ_c, v_c, and r_h are $M_\odot\,\mathrm{pc}^{-3}$, $\mathrm{km\,s}^{-1}$, and pc, respectively.

4. SUMMARY

We have investigated the evolution of isolated two-component clusters with initial condition of Plummer models. The corecollapse time t_{cc} showed a good correlation with a parameter $(N_1/N_2)^{1/2}(m_1/m_2)^2 t_{rh}$. To explain this correlation, a new time scale for the precollapse evolution of two-component clusters, $t_{r2} \equiv (N_1/N_2)^{2/3}(m_1/m_2)^{5/3} t_{rh}$ have been introduced using Spitzer's (1969, 1987) global equipartition analysis.

We also found two-component clusters which best match with our multi-component clusters with power-law mass functions. For example, the evolution of 11-component cluster with a Salpeter mass function and $M = 10^5\,M_\odot$ was well described by a two-component cluster with $m_2/m_1 = 2.2$, $N_1/N_2 = 21$, $N = 1.3 \times 10^5$, and $M = 0.84 \times 10^5\,M_\odot$. Furthermore, it has been found that the best matching two-component cluster has N_2 very close to the number of stars heavier than turnoff mass of the multi-component cluster.

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